Mastery in Mathematics

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MASTERY IN MATHEMATICS

What does it mean to understand in mathematics?

Consider someone you know that you would identify as having mastery of mathematics, why do you choose this person?

What is it that this person can do that makes them 'mathematical' or seem good at mathematics?

Mastery in Mathematics

- A deep conceptual understanding
- An ability to reason and explain
- Ability to understand and use mathematical vocabulary
- Flexibility and resilience when approaching problems
- Fluency and efficiency
- An ability to solve problems in a range of contexts

- *Requires a particular ethos and identity*
- *Requires opportunities to overlearn*
- *Requires collaboration*
- *Places emphasis on discussion and proof*
- *Pupils move at broadly the same pace*

Assumptions/discourses about ability in mathematics

"You can either do maths or you can't"

"Only some people are good at maths"

"You are either mathematical/scientific or arty/creative"

"I was never any good at maths" (parent to child)

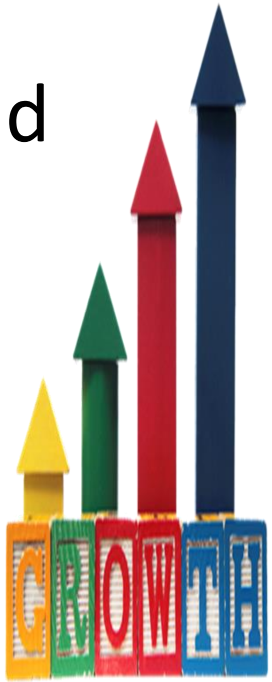
"When will I ever use this in everyday life?"

Can you think of any others?

Growth mindset (v. fixed mindset)

“Do people with this mindset believe that anyone can be anything, that anyone with proper motivation or education can become Einstein or Beethoven? No, but they believe that a person’s true potential is unknown (and unknowable); that it’s impossible to foresee what can be accomplished with years of passion, toil, and training.”
(Dweck, 2006)

We need to allow ‘different rates of the developmental trajectory’ (Stigler, 2011)



Charlie Stripp's (Head of NCETM) Blog

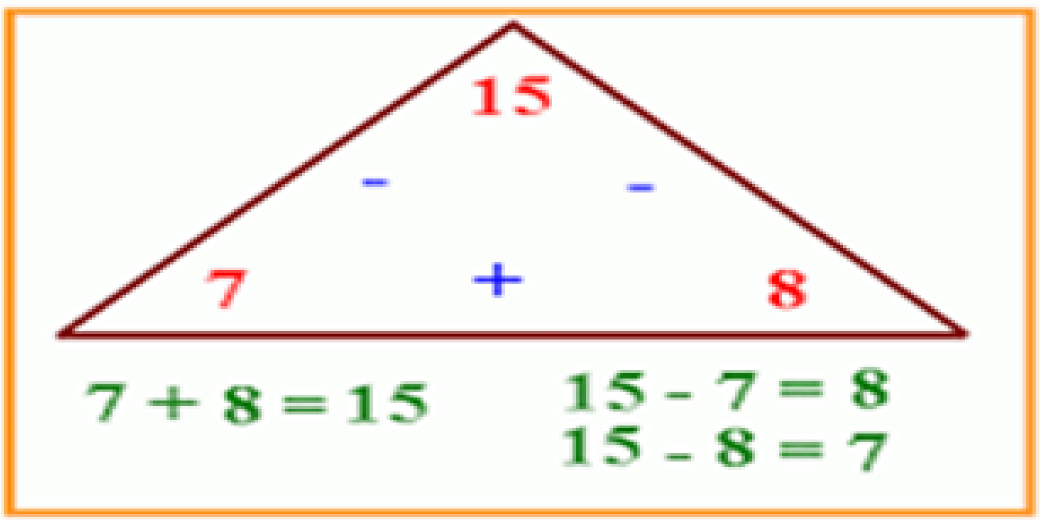
"I think it may well be the case that one of the most common ways we use differentiation in primary school mathematics… has had, and continues to have, a very negative effect on the mathematical attainment of our children at primary school and throughout their education."

- Labels and ideas of smartness and giftedness carry with them fixed ideas about ability, suggesting to students that they are born with a gift or a special brain. When students are led to believe they are gifted, or they have a “math brain” or they are smart and later struggle, that struggle is absolutely devastating. Students who grow up thinking that they have a special brain often drop out of STEM subjects when they struggle.

- Jo Boaler (2017)

What is meant by mastery and depth?

Consider what you would expect a child to be able to do if they had a deep understanding of number facts?

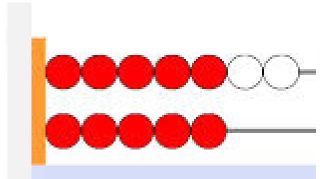


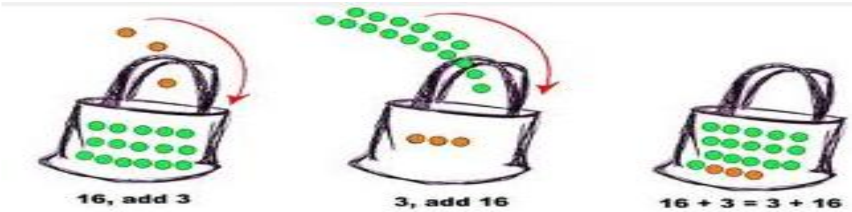
<u>What is ‘mastery’ of number bonds?</u>

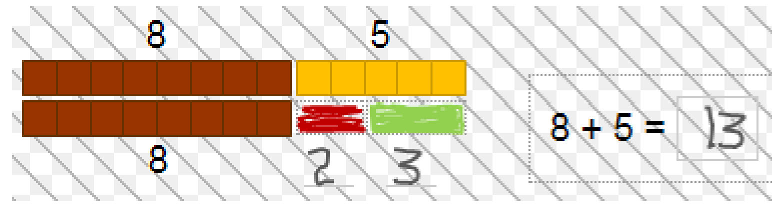
Mastery of Number Bonds

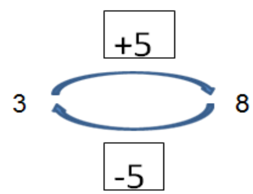
- **Memorising facts**
- **Using facts**
  - Deriving new facts
  - Bridge
- **Laws and principles**
  - Inverse relationships
  - Commutativity
  - Associativity
- **Patterns and connections**
  - Modelling with apparatus
  - Comparing apparatus
  - Explanation
  - Same different/ what do you notice
  - Own examples
- **Generalising**

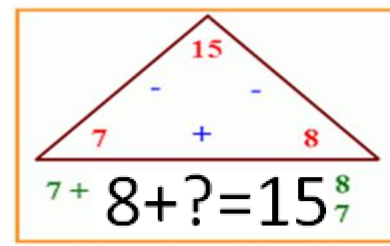
'How many ways can you make 7?'

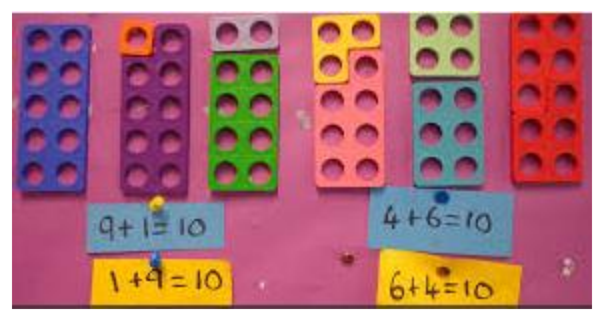


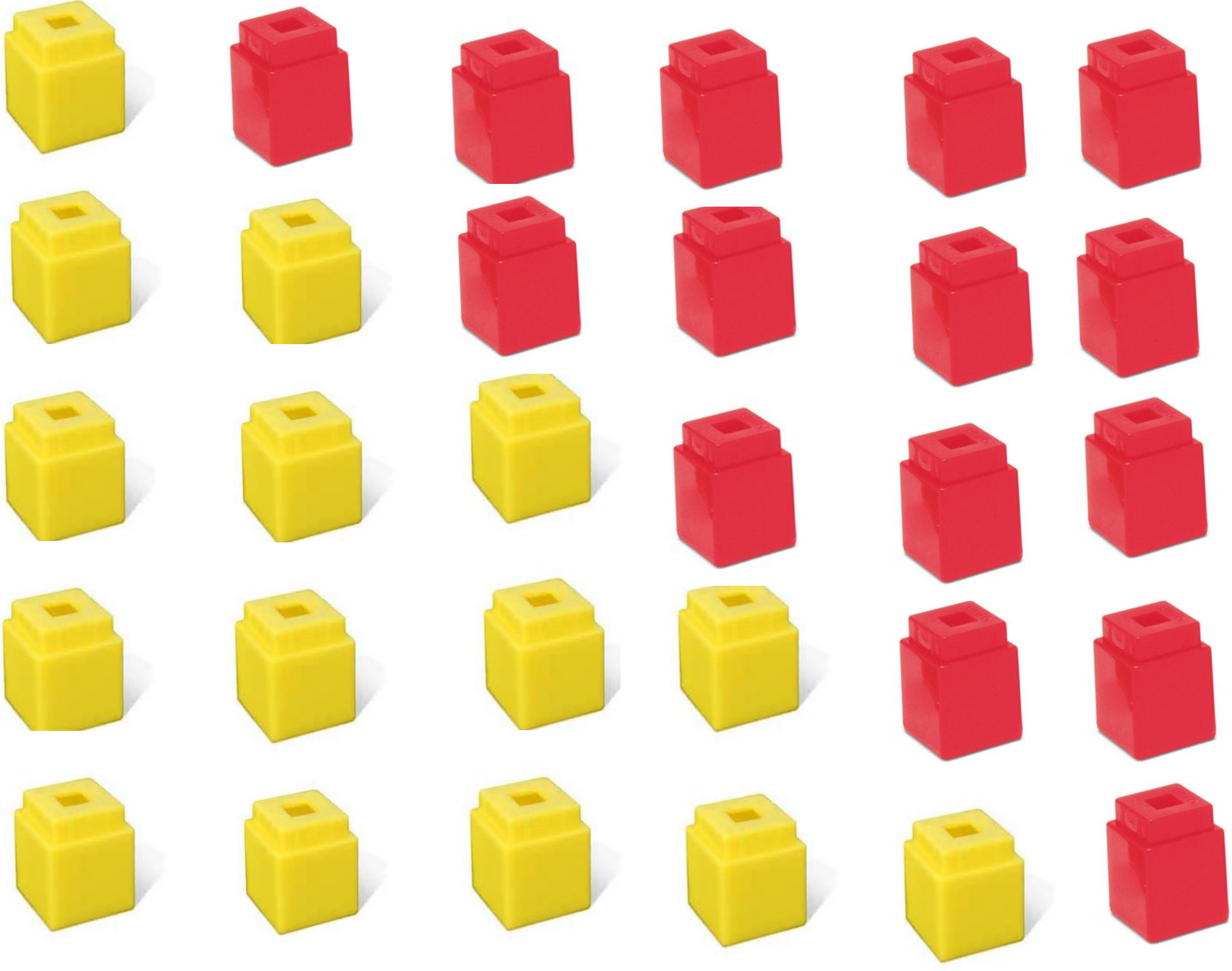












Whole	Part	Part
5	5	0
5	4	1
5	3	2
5	2	3
5	1	4
5	0	5

$\square + 3 = 5$

$5 = 3 + \square$

$\square - 2 = 3$

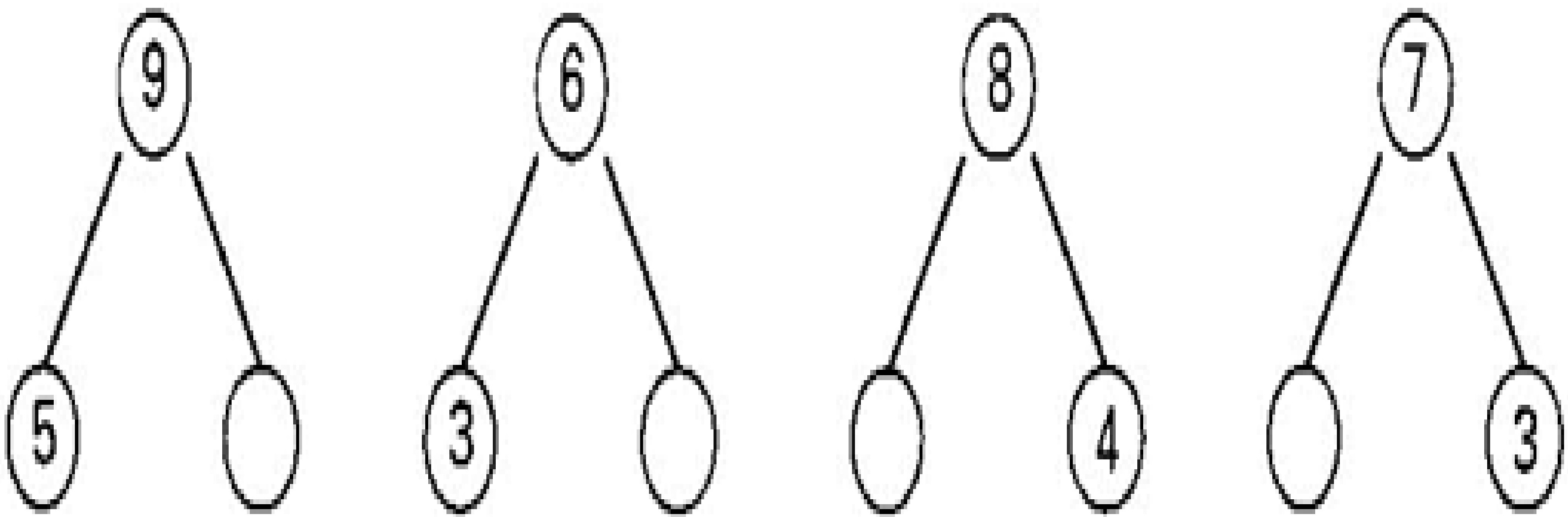
$\square - 3 = 2$

$2 + \square = 5$

$3 = 5 - \square$

Learning facts as a trio

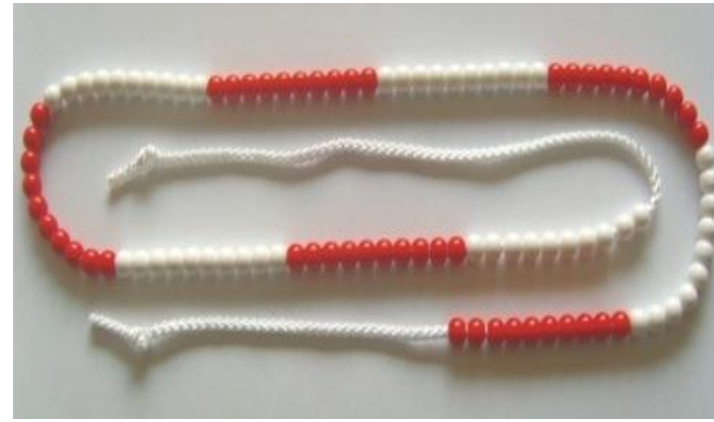
Teachers can use the triad format in simple written exercises, such as 'fill in the missing numbers'.



9-?=4 5+?=9 ?-5=4 9=4+? ...

Patterns, relationships and structures

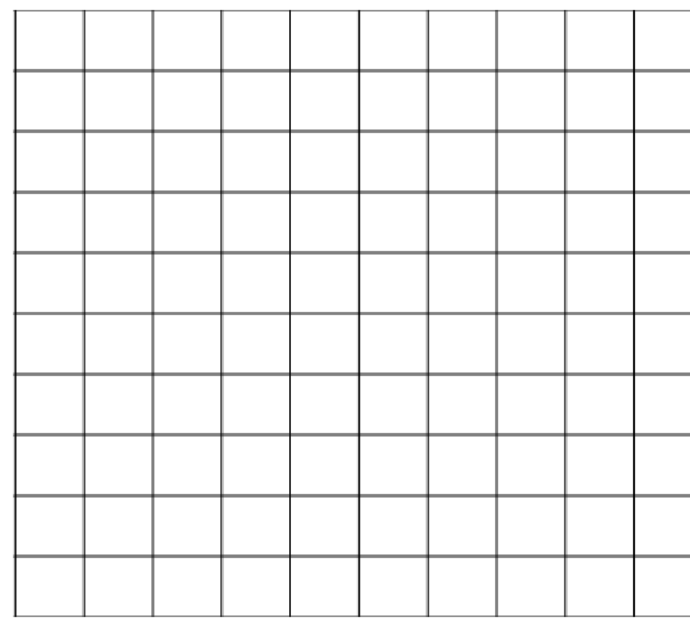
- 2+3, 12+3, 22+3, 32+3
- 6+?=10, 16+?=20, 26+?=30
- 10-1, 20-1, 30-1



What do you notice?

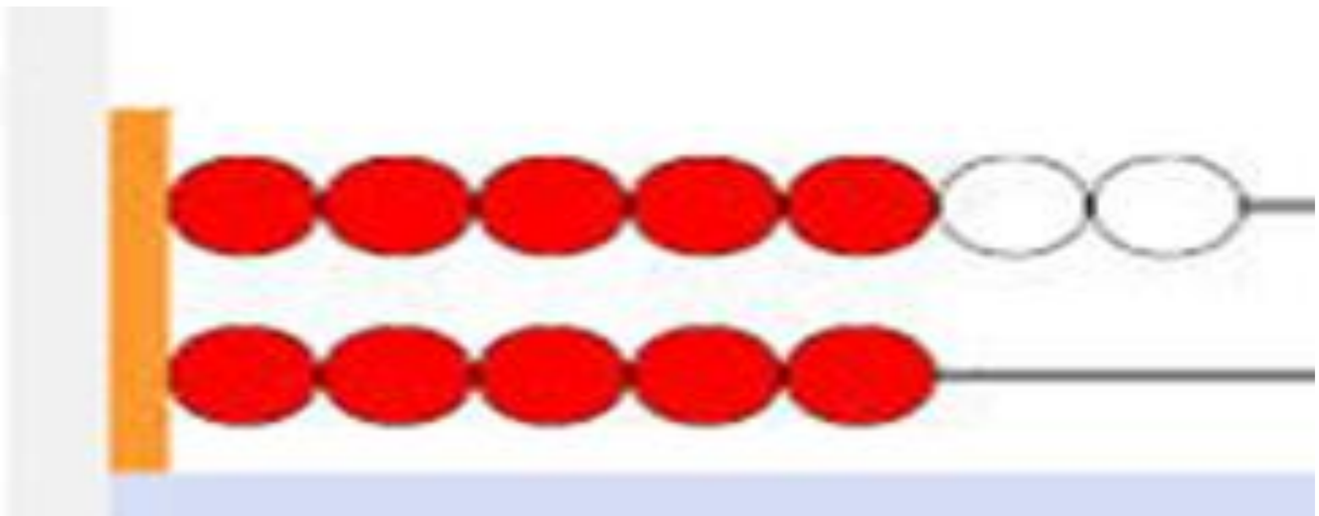
Will it always happen?

Why does it keep happening?



Patterns, relationships and structures

If you know 5+5=10

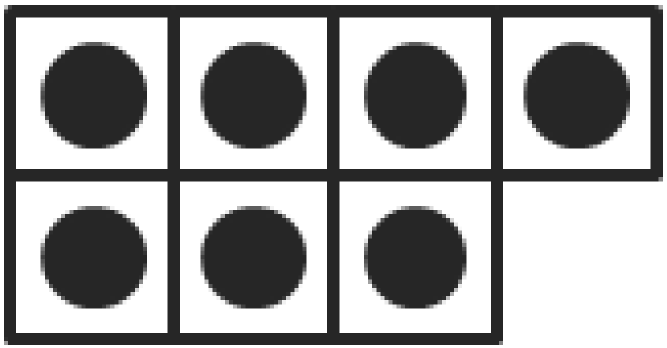


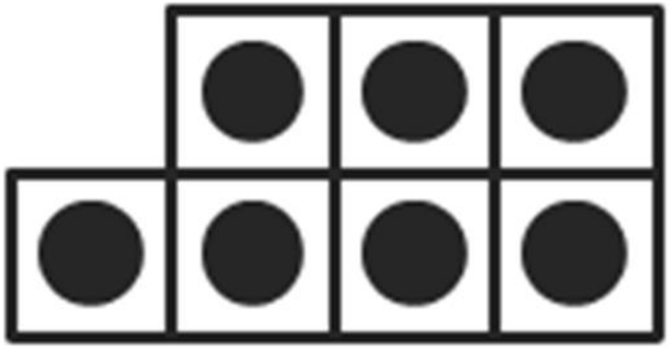
What else do you know?

Bridging through multiples of 10

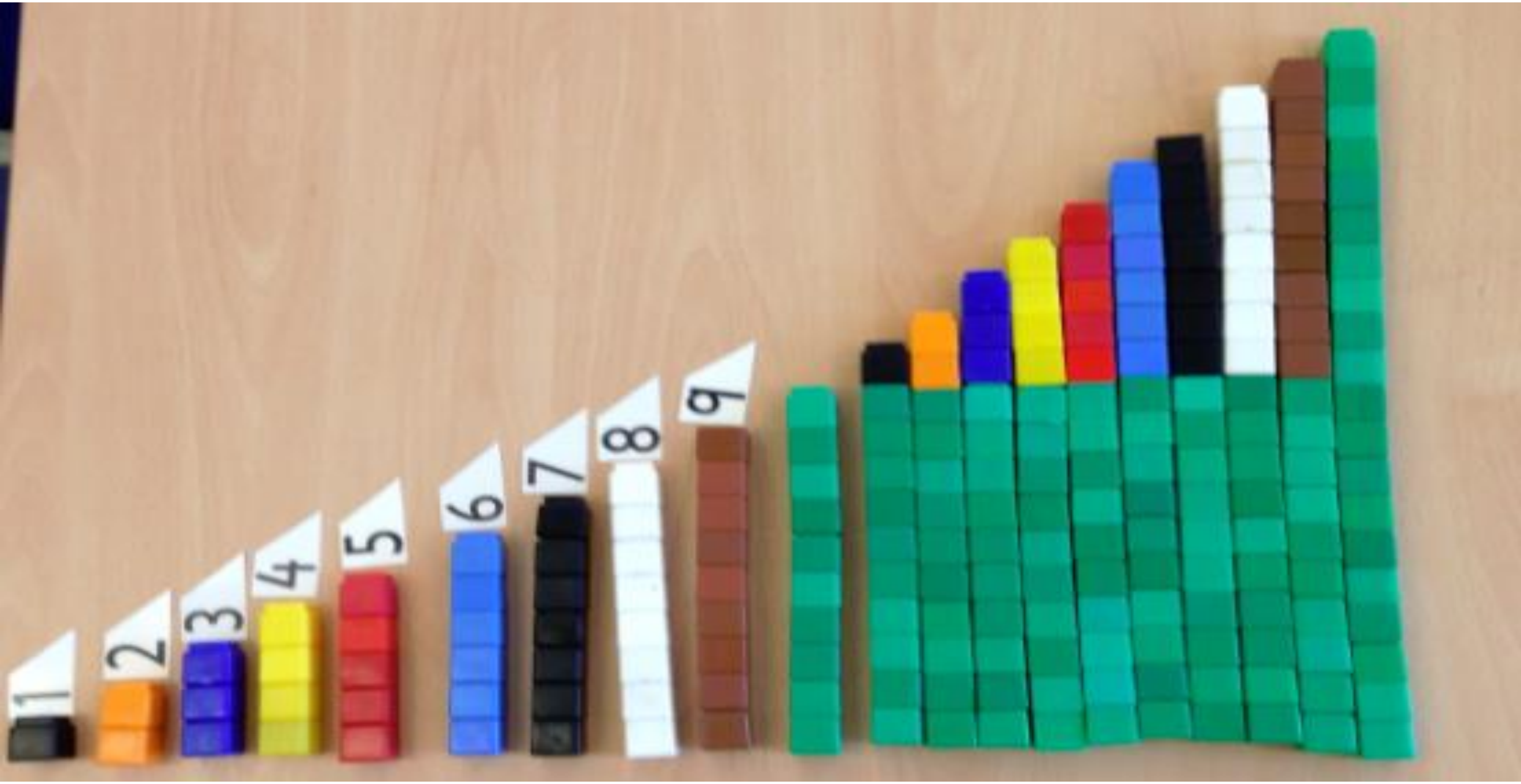
Visualising a structure or a relationship

What happens when we add two odd numbers?

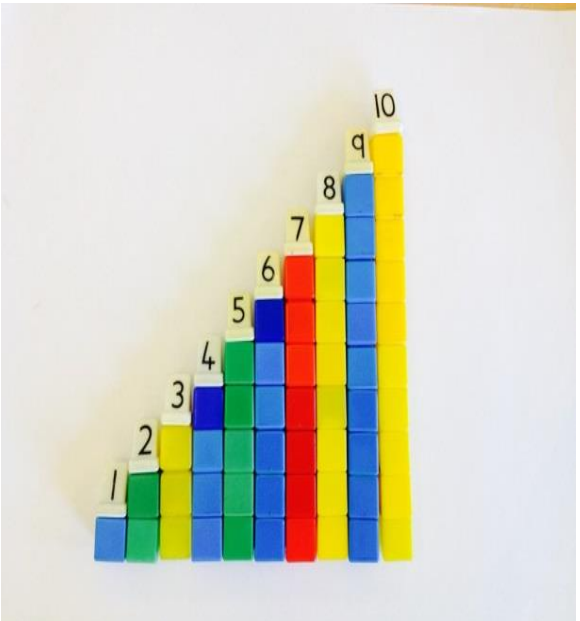


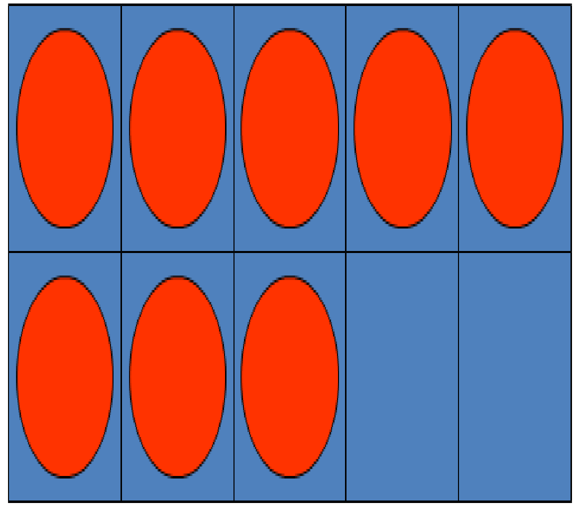


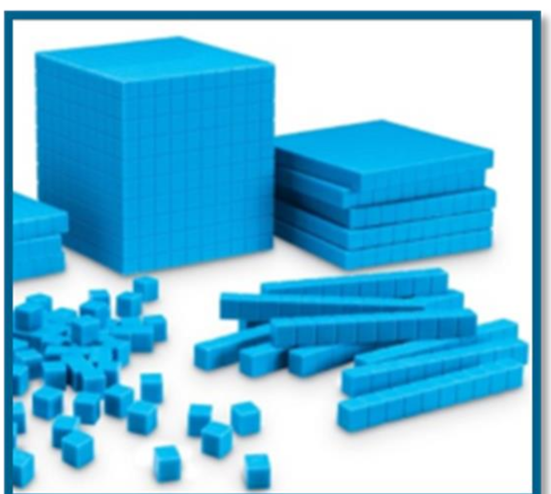
Resources and Structures



Link resources, contexts and models

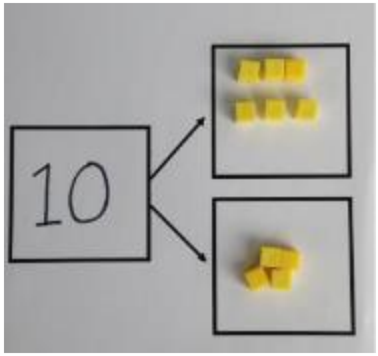


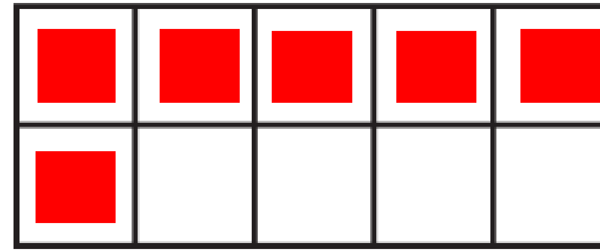


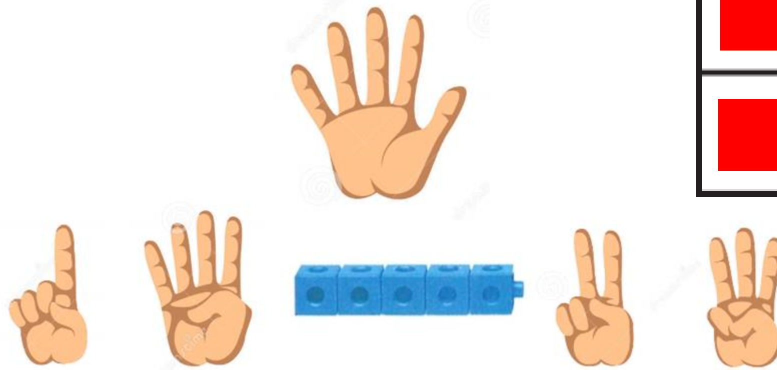


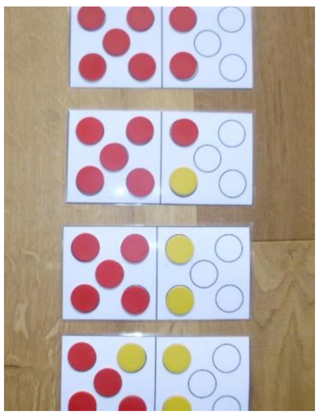
Here 13 x 5 = (10 x 5) + (3 x 5).

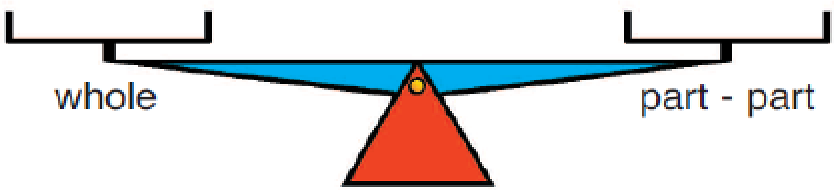
Some more models…

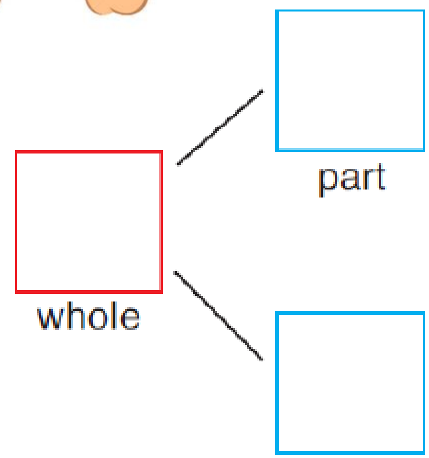




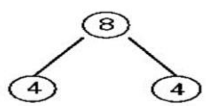








$$10 = 6 + 4$$
$$10 - 6 = 4$$
$$10 - 4 = 6$$
$$10 = 4 + 6$$



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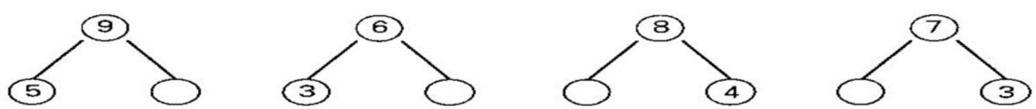
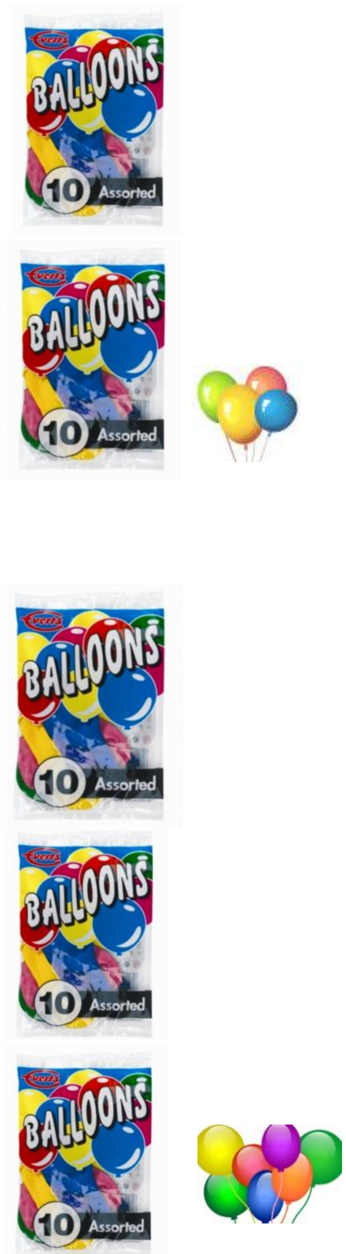
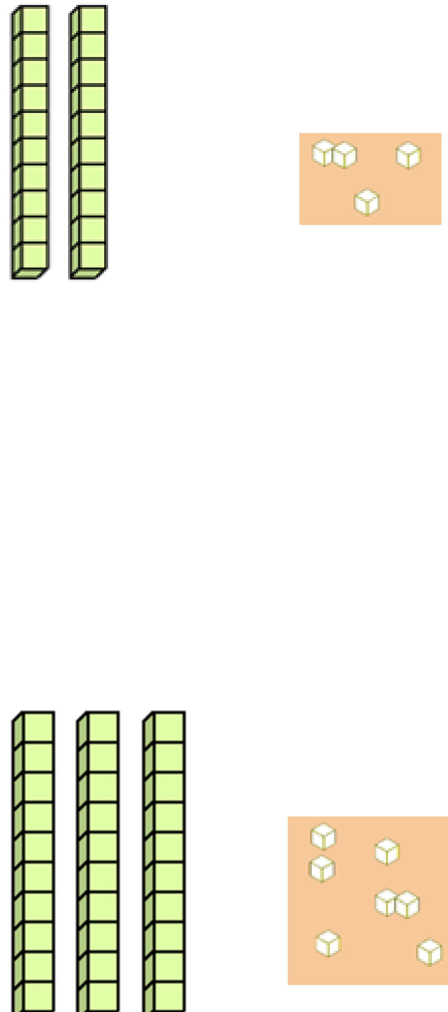
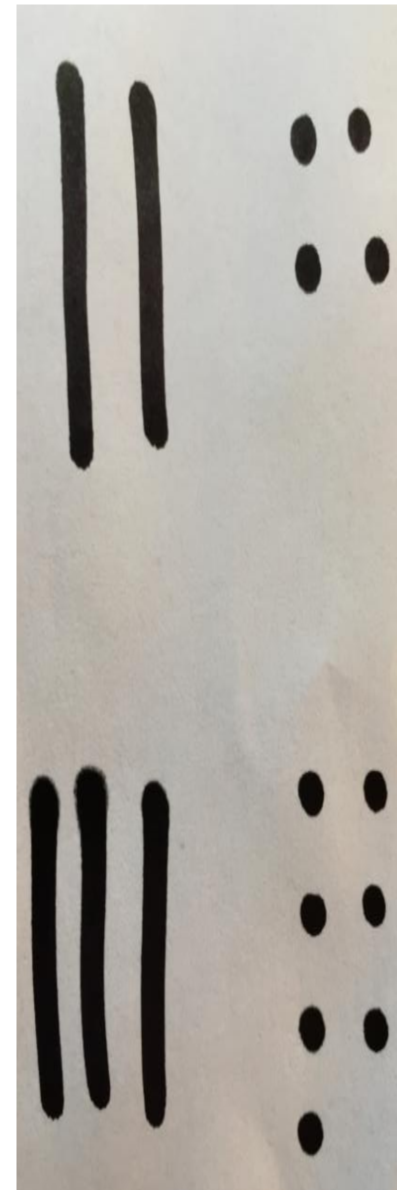


Models, Language, Recording, Visualising

24+37

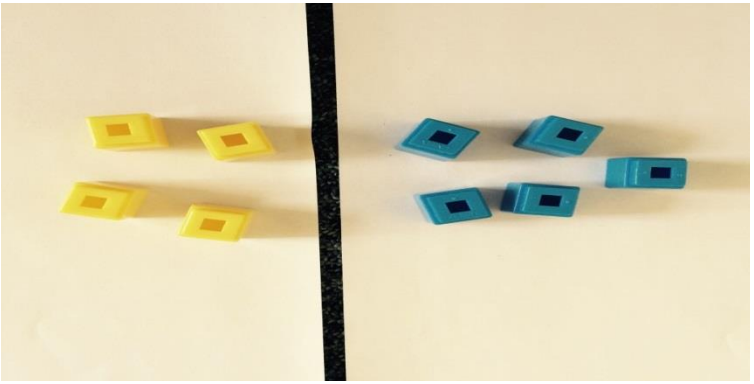






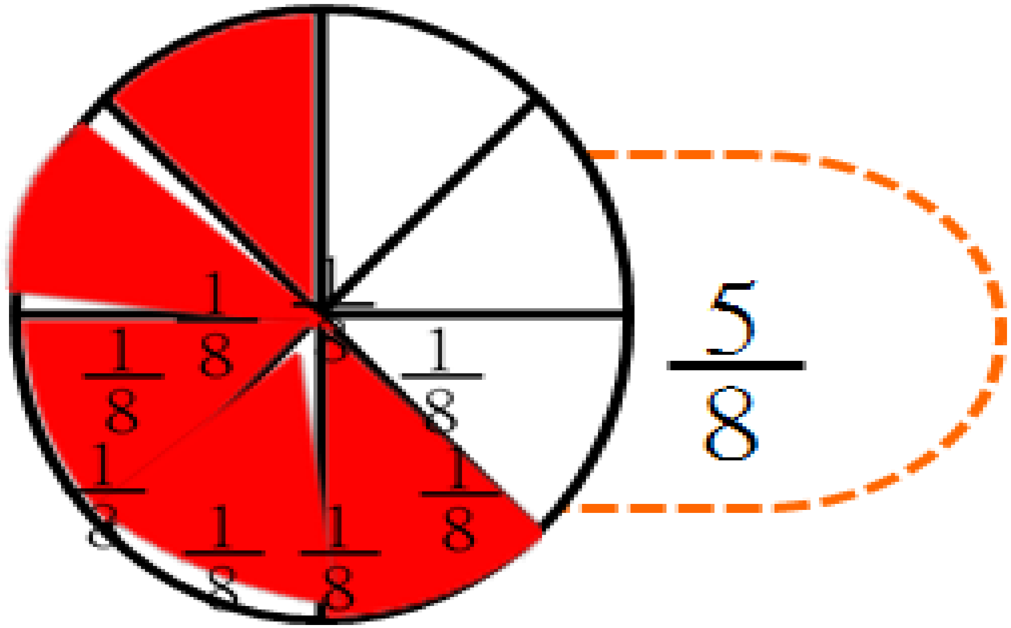

How Many 1ps ?

Developing Explanation



One, two, three, four, five. There are five cubes here.
One, two, three, four. There are four cubes here.
One, two, three, four, five, six, seven, eight, nine. There are nine cubes altogether.
Five plus four is equal to nine.

Write down the fractions to show the red parts as a fraction of the circle.



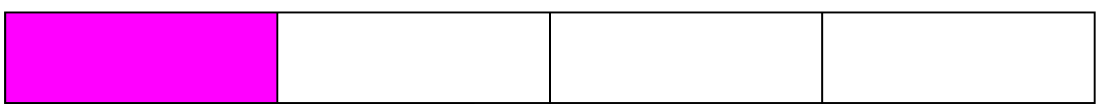
The whole is divided into ___ equal parts. Each part is worth _____ of the whole. If I take ___ of the parts then this is ___ times _____ of the whole. This is ___.

Reasoning NOT Rushing

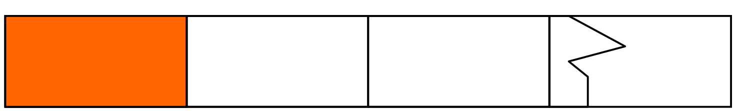
2 paper tapes were broken, can you guess which original paper tape is longer?

Why? How do you get your answer?

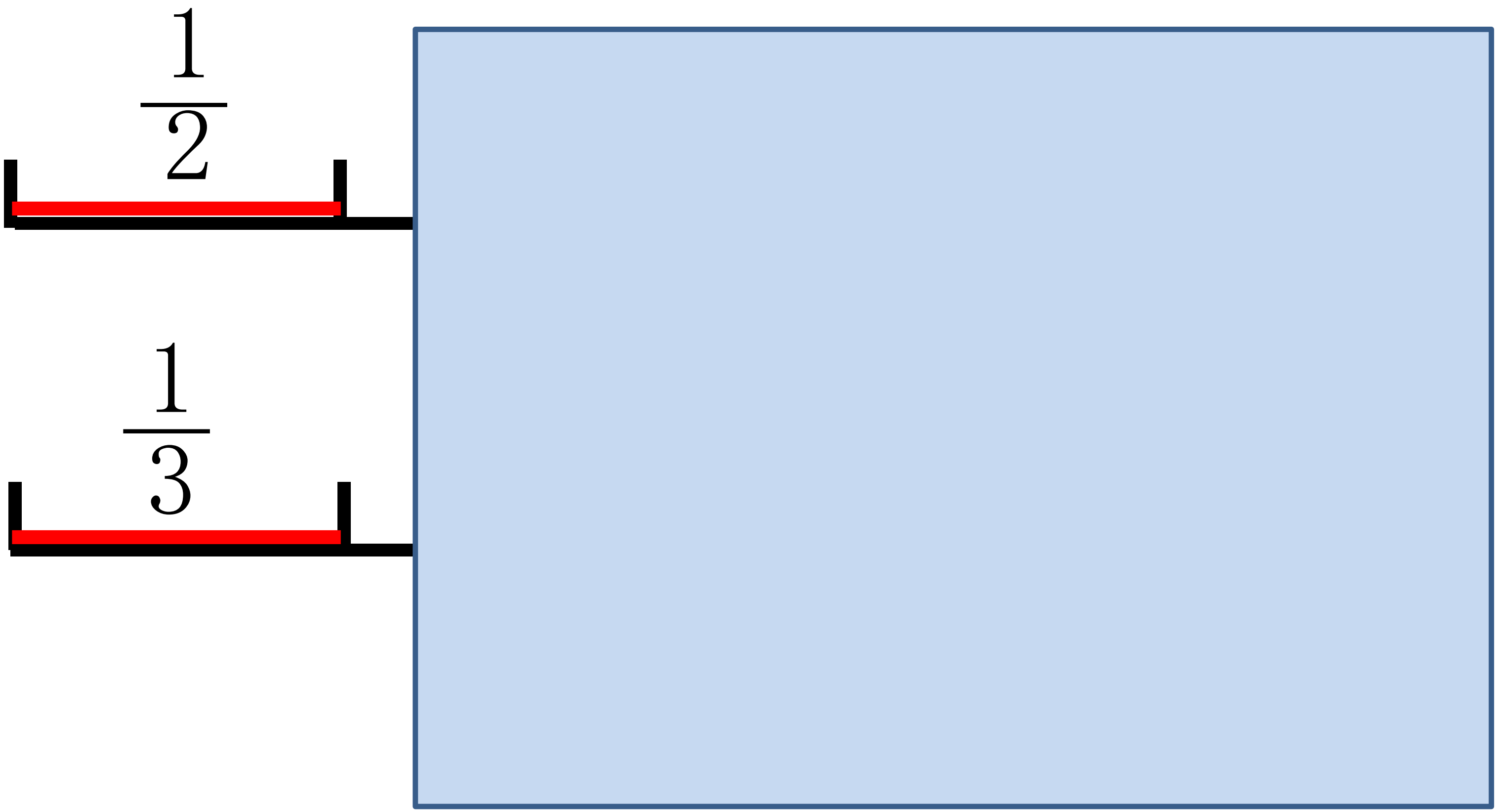
$\frac{1}{5}$



$\frac{1}{5}$



Think: Which line is longer?



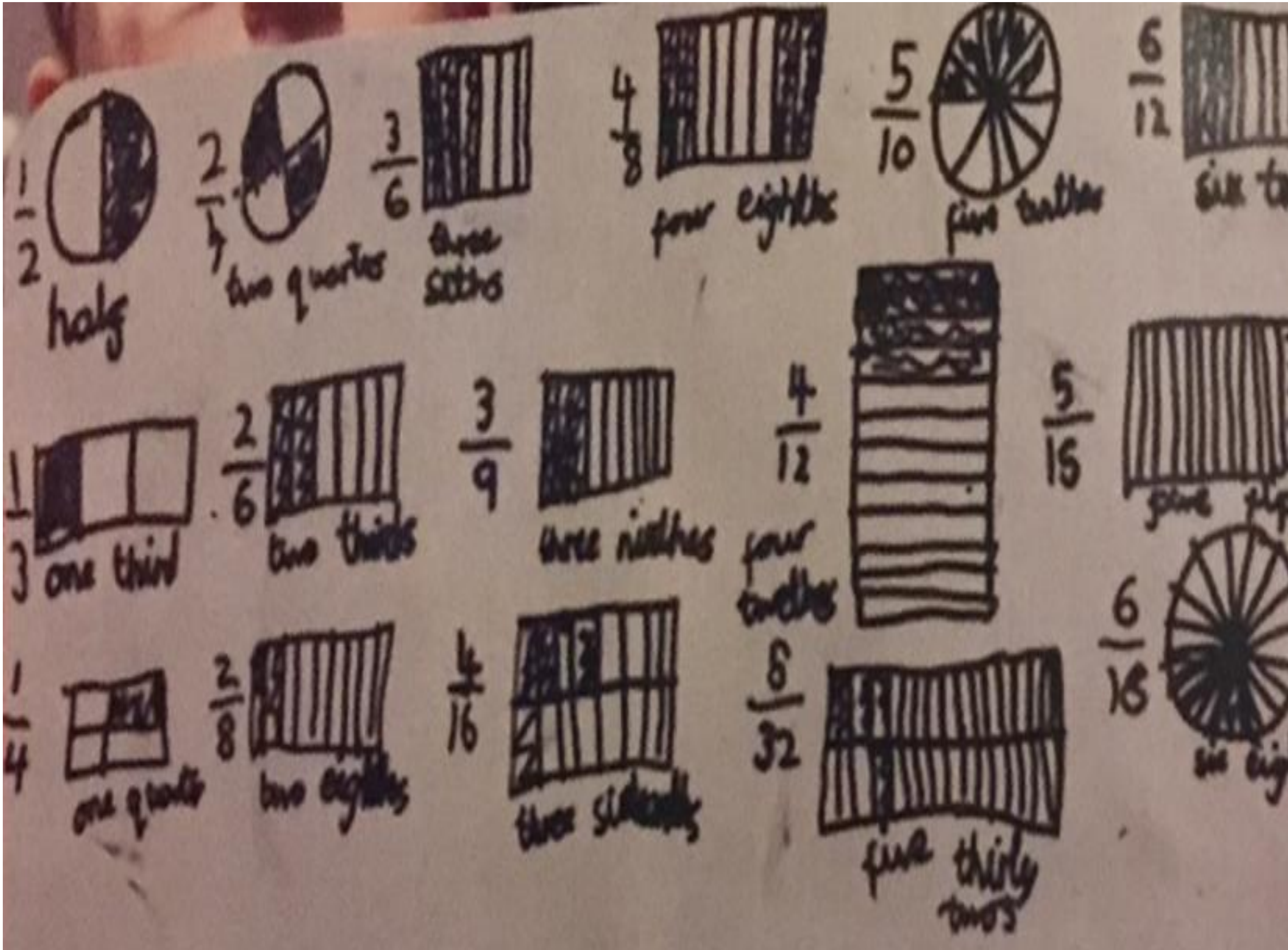
First:

Second:

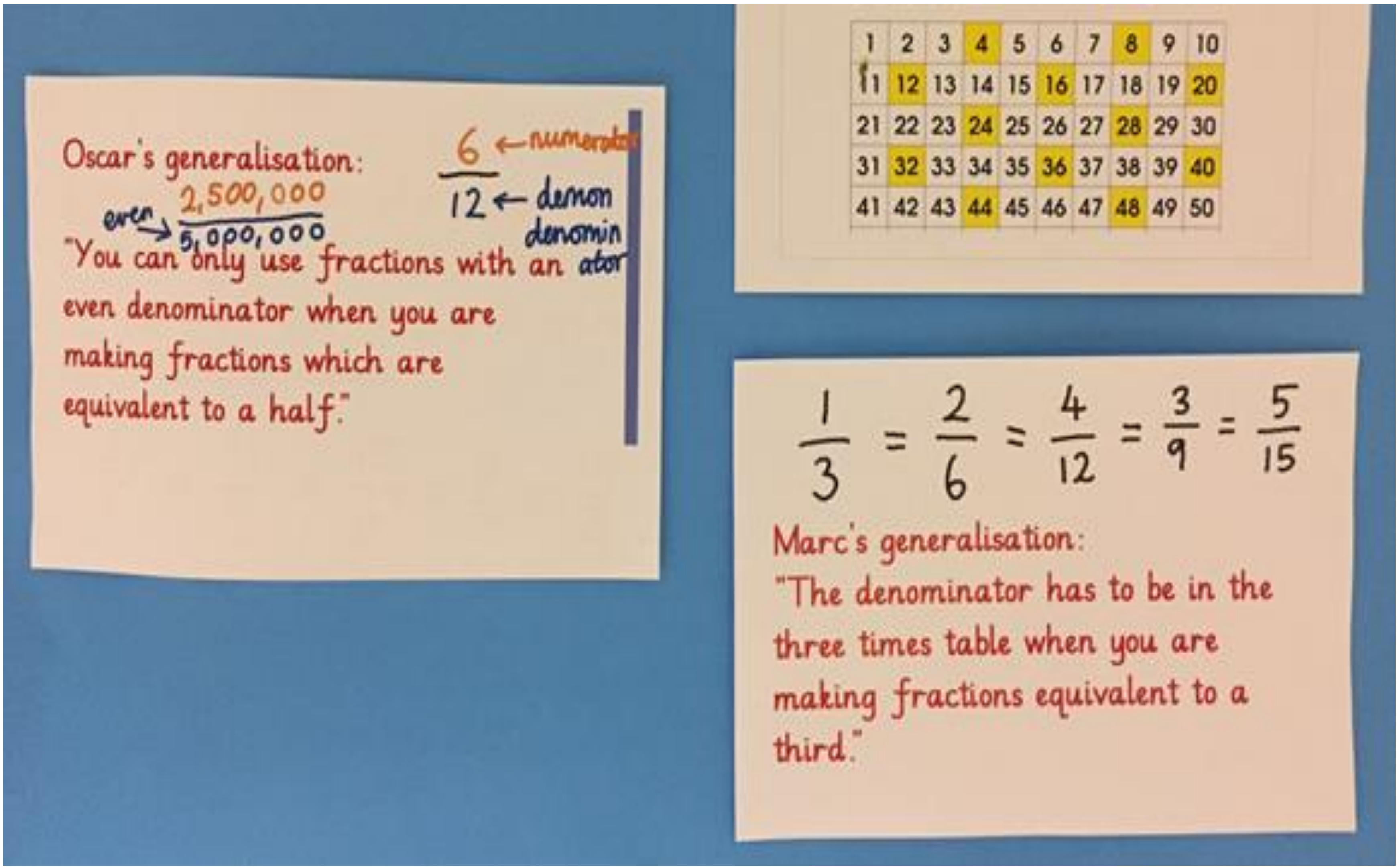
How might you calculate?

$$4 \times 376 = 2 \times ?$$

Providing a model to explain or prove


1/2
half
2/4
two quarters
3/6
three sixths
4/8
four eighths
5/10
6/12
1/3
one third
2/6
two thirds
3/9
4/12
four twelths
5/15
1/4
one quarter
2/8
two eighths
4/16
8/32
five thirty twos
6/16

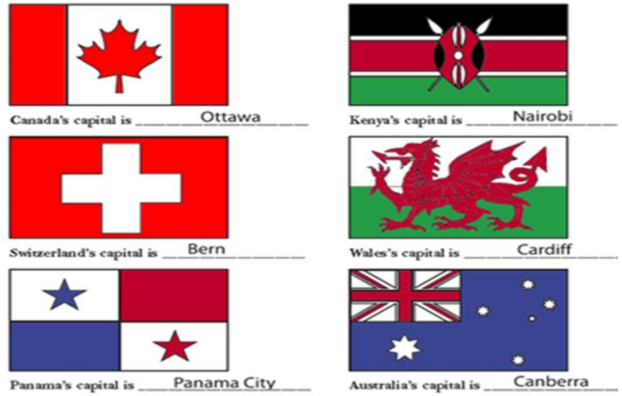
Generalising about Equivalence

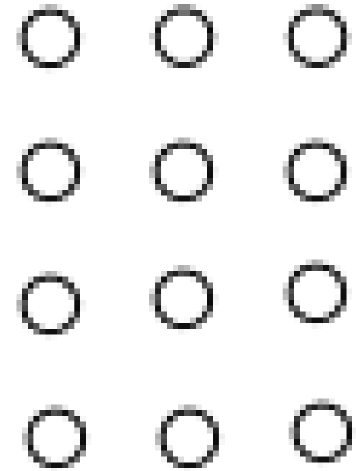


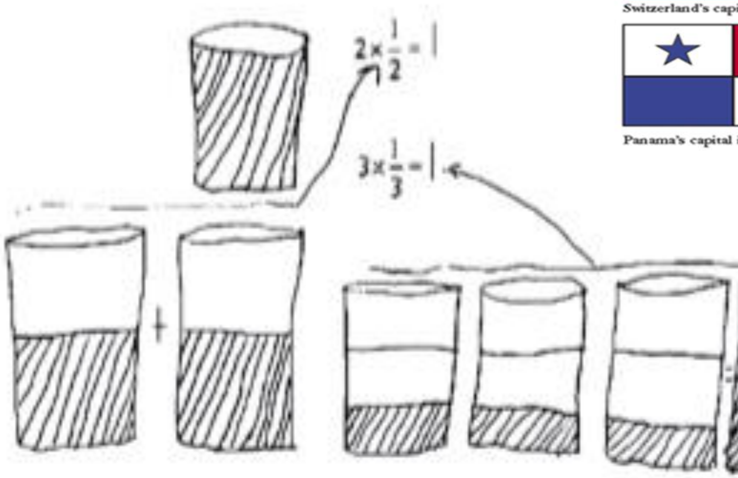
Rich Tasks NOT Rushing

I have 36 chocolates, how many different rectangular boxes can you suggest to the manufacturer?

Multiple Models: Variation







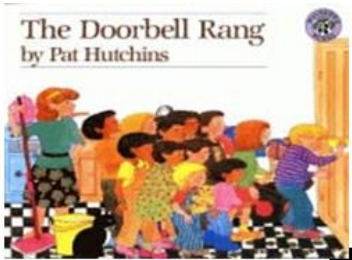
Fractions

'*No area of elementary mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions, ratio and proportionality*' Smith (2002)

Equal Sharing and fractions as an extension of sharing with whole numbers

- 6 biscuits between 2 then 3 children etc
- 3/5/7 biscuits between 2 children

- 5 biscuits between 4 children
- 4 biscuits between 3 children



- 5 biscuits between 3 children

- 7 biscuits between 4 children

Equal Sharing

4 cookies 3 kids

- Joe: How would you share them?
- Leah: One, one, one and there is one left. Then they each get one third, one third, one third
- Joe: So how much does each kid get?
- Leah: They get one whole one and one third

5 cookies and 3 kids

- Leah: One, one, one and there is two more left. They get a third, a third, a third and then a third, a third, a third
- Joe: So how much do they each get?
- Leah: They get one and two thirds

7 cookies shared between 4 kids

Leah: That's a hard one, maybe I can't do it.

Joe: Think about what you did to solve the other two.

Leah: Whole, whole, whole, whole, then there's three more left. Um, three more cookies left. Then you break up one into halves, then there are two left. And, another into half, half. Break the last one into quarter, quarter, quarter, quarter.

Joe: Great! How much does each kid get?

Leah: One whole, one half and one quarter.

Leah is four.

(Anthony and Walshaw, 2007)

Contrast this with my daughter's homework in year 2…

Mastery versus Acceleration

Year 2 Octagon Group: Find 15/17 of 51

Me: What's this '½' mean?
Daughter: I don't have a clue.

We then talked about:

- One part out of two
- Half

And then we

- walked half way down the garden
- half way up the stairs
- chopped a potato in half ….

What can we do as parents

- Be a maths enthusiast
- Get them to talk about the maths they have done each day
- Get them to explain the maths to you
- Support them through practice with memory aspects of maths:
  - Vocabulary
  - Facts
  - Procedures
- Encourage them to make connections
  - between different aspects of maths
  - real life maths
- Go shopping with real money and play games

Any Questions?